DETAILS EXPLANATIONS

CE : Paper-1 (Paper-5) [Full Syllabus]

[PART : A]

1. Various stresses are :
- Hoop stress
- Longitudinal stress
- Radial Stress

2. $$\sigma = \frac{270\times10^6}{\frac{200\times400^2}{6}} = 50.625 \text{ N/mm}^2$$

3. It is the number of unknown in a static truss of frame that can't be calculated using equilibrium equations only.

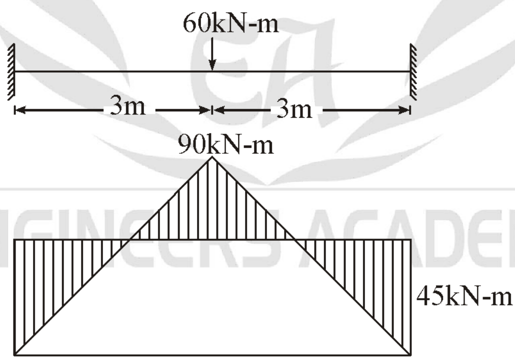
4. It is used to counter shear, creep and temperature effects.

5.
- Friction
- Slip of anchor
- Creep of concrete

6. Coefficient of permeability of soil is equal to the ratio of velocity through soil to the hydraulic gradient.

$$k = \frac{V}{i}$$

7. ***B.M.D. :***



8. It is the process by which the rocks get converted in soil by crumbling due to weathering.

9. $$x_{u_{lim}} = 120 \text{ mm} = k.d$$

$$d = \frac{120}{0.53} = 226.41 \text{ mm}$$

10. Limiting depth of neutral axis :

$$x_{u_{lim}} = kd = 0.48 \times 300 = 144 \text{ mm}$$

11. It can be uninstalled and again installed.

12. Ratio of lateral strain to longitudinal strain is called 'poisson's-ratio'.

13. $M_{AB} = M_{FAB} + \frac{3EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$

14. Vertical stiffners are secondary plates or sections which are attached to beam webs or flanges to stiffen them against out of plane deformations.

15. Shape-factor is the ratio of plastic section-modulus to the elastic section modulus.

16. For bi-axial loading :

$$q = \text{Shear stress} = 0$$

$$\text{radius} = r = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

17. Distribution factor distributes the moment applied to any junction, to the members connecting at that junction.

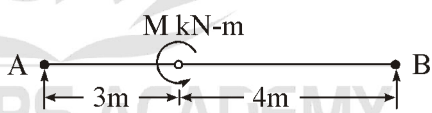
18. In double cover butt joint there are two shear planes, so shear capacity is double.

19. $\frac{l}{l.l.D} \ngtr 12$

20.
- By increasing grade of concrete.
- By increasing grade of steel.

[PART : B]

21. For hinged support :



$$\Sigma M_A = 0$$

$$R_B.(3+4) = M$$

$$\because \quad \Sigma V = 0$$

$$R_B = \frac{M}{7}(\uparrow)$$

$$\therefore \quad R_A = \frac{M}{7}(\downarrow)$$

22. Flexural Rigidity = EI

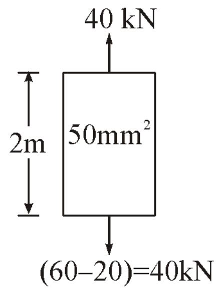
$$E = \text{For mild steel} = 2 \times 10^5 \text{ N/mm}^2$$

$$I = \frac{BD^3}{12} = \frac{200 \times 300^3}{12} = 450000000 \text{ mm}^4$$

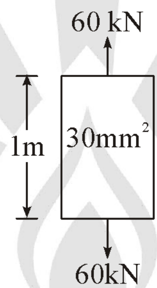
$$EI = 2 \times 10^5 \times 450000000$$

$$EI = 9 \times 10^{13} \text{ N-mm}^2$$

23. By superposition method :



$$\Delta_1 = \frac{P_1 l_1}{A_1 E_1} = \frac{40\times10^3\times2000}{50\times1\times10^5} = 16 \text{ mm}$$



$$\Delta_2 = \frac{P_2 l_2}{A_2 E_2} = \frac{60\times10^3\times1000}{30\times1\times10^5} = 20 \text{ mm}$$

Total deflection = $\Delta_1 + \Delta_2$ = 16 + 20 = 36 mm

24. Maximum bending moment at center :

$$\frac{wl}{4} = 20 \Rightarrow \frac{w\times8}{4} = 20 \Rightarrow w = \frac{20\times4}{8} = 10 \text{ kN}$$

25. For double cover butt joint there are two shear planes :

$$v_{dsb} = n \times A_{sb} \times f_{ub} \times \frac{1}{\sqrt{3}\,\gamma_{mb}}$$

$$= 2 \times 0.78 \times \frac{\pi}{4} \times \frac{20^2\times400}{\sqrt{3}\times1.25}$$

$$= 90545 \text{ N} = 90.55 \text{ kN}$$

$$\gamma_{mb} = 1.25$$

26. $A_{net_1} = 140 \times 10 - (17.5 \times 10) = 1225 \text{ mm}^2$

$$A_{net_2} = 140 \times 10 - (2 \times 17.5 \times 10) + \frac{(25)^2 \times 10}{4 \times 70}$$

$$= 1072.32 \text{ mm}^2$$

$A_{net} = 1072.32 \text{ mm}^2$

27. $M_{u_{lim}} = 0.36 f_{ck} B x_{u_{lim}} (d - 0.42 x_{u_{lim}})$

$$= 0.36 \times 20 \times 300 \times (0.48 \times 400)\{400 - (0.42 \times 0.48 \times 400)\}$$

$M_{u_{lim}} = 132.38 \text{ kN-m}$

28. The minimum steel provided as longitudinal reinforcement is = 0.8% of gross area.

$$A_{sc_{min}} = \frac{0.8}{100} \times \frac{\pi}{4} \times 600^2 = 2261.95 \text{ mm}^2$$

29. Distribution factor (D.F):

For member AB :

$$\frac{3EI}{L} = \frac{3 \times E \times I}{4} = \frac{3EI}{4}$$

For member BC :

$$\frac{4EI}{L} = \frac{4EI}{5}$$

$$(D.F.)_{AB} = \frac{3EI/4}{\left(\frac{3EI}{4} + \frac{4EI}{5}\right)} = 0.483$$

$\Rightarrow$ (D.F.) = 1 – 0.483 = 0.516

30. Shrinkage Ratio

$$R = \frac{(V_1 - V_d)/V_d \times 100}{(w_1 - w_s)}$$

Taking $V_1 = V_L = 40$ cc and $w_1 = w_l = 60\%$

$$R = \frac{(V_L - V_d)/V_d \times 100}{(w_L - w_s)} = \frac{(40 - 23.5)/23.5 \times 100}{(60 - 20)} = 1.75$$

31. Total volume V = 50 cm^3

Water is displaced by solids, so, V_s = 25 cm^3

$\therefore$ Volume of voids = 50 – 25 = 25 cm^3

$$\therefore \text{ Void Ratio } e = \frac{V_v}{V_s} = \frac{25}{25} = 1 = 100\%$$

$$\text{Porosity } n = \frac{V_v}{V} = \frac{25}{50}$$

$$= \frac{1}{2} = 50\%$$

32. Coefficient of uniformity

$$\Rightarrow \quad C_u = \frac{D_{60}}{D_{10}} = \frac{1000\mu}{100\mu} = 10$$

Coefficient of Curvature

$$C_c = \frac{D_{30}^2}{D_{10}.D_{60}} = 0.9$$

[PART : C]

33. Analysing the frame by slope deflection method which is a stiffness method.

Fixed end moments :

$$M_{FAB} = 0 \quad ; M_{FBA} = 0$$

$$M_{FBC} = -\left(\frac{10\times 5^2}{12} + \frac{20\times 5}{8}\right)$$

$$= -33.33 \text{ kN-m}$$

$$M_{FCB} = +33.33 \text{ kN-m}$$

Slope Deflection Equations :

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{AB} = 0 + \frac{2EI}{4}(2\times 0 + \theta_B - 0)$$

$$M_{AB} = 0.5EI\theta_B$$

and
$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$M_{BA} = 0 + \frac{2EI}{4}(2\theta_B + 0 - 0)$$

$$M_{BA} = EI\theta_B$$

and

$$M_{BC} = M_{FBC} + \frac{2E(2I)}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$M_{BC} = 33.33 + \frac{4EI}{5} + (2\theta_B + \theta_C)$$

$$M_{BC} = -33.33 + 0.8EI(2\theta_B + \theta_C)$$

$$M_{CB} = M_{FCB} + \frac{2E(2I)}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{CB} = +33.33 + \frac{4EI}{5}(2\theta_C + \theta_B)$$

$$M_{CB} = 33.33 + 0.8EI(2\theta_C + \theta_B)$$

Equilibrium Equation at Joints :

(i) $M_{BA} + M_{BC} = 0$

$$EI\theta_B - 33.33 + 0.8EI(2\theta_B + \theta_C) = 0$$

$$2.6\theta_B + 0.8\theta_C = \frac{33.33}{EI} \quad \text{...(1)}$$

(ii) $M_{CB} = 0$

$$0.8EI(\theta_B + 2\theta_C) + 33.33 = 0$$

$$0.8\theta_B + 1.6\theta_C = \frac{-33.33}{EI} \quad \text{...(2)}$$

By solving equation (1) and (2)

$$\theta_B = \frac{22.725}{EI}$$

$$\theta_C = \frac{-32.194}{EI}$$

From equation (1) and (2)

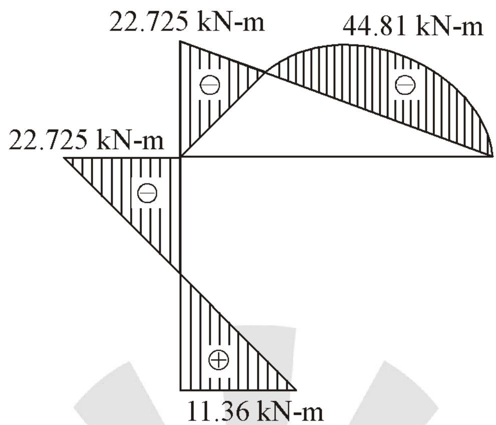
$$M_{AB} = 11.36 \text{ kN-m}$$

$$M_{BA} = 22.75 \text{ kN-m}$$

$$M_{BC} = -22.725 \text{ kN-m}$$

$$M_{CB} = 0$$

B.M.D. :



Calculating reactions :

$$V_A = 39.545 \text{ kN}$$
$$H_A = H_C = 8.52 \text{ kN}$$
$$V_C = 30.455 \text{ kN}$$

34. When a shaft is subjected to both bending and torsion, the magnitude of principal stress is given by :

$$\frac{\sigma_1}{\sigma_2} = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\sigma_1 = \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\sigma_2 = \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$$

$$\therefore \quad (\sigma_1 - \sigma_2) = 2\left(\frac{16}{\pi D^3}\sqrt{M^2 + T^2}\right)$$

$$\sigma_1 - \sigma_2 = \frac{32}{\pi D^3}\sqrt{M^2 T^2}$$

According to maximum shear stress theory :

$$\sigma_1 - \sigma_2 \leq \frac{\sigma_y}{F.O.S.}$$

$$\Rightarrow \quad \frac{32}{\pi D^3} \times \sqrt{M^2 + T^2} \leq \frac{\sigma_y}{F.O.S.}$$

$$\Rightarrow \quad \frac{32}{\pi D^3} \times \sqrt{(20 \times 10^6)^2 + (40 \times 10^6)^2} \leq \frac{250}{2.0}$$

$$\Rightarrow \quad D^3 \geq 3644224.22$$

$$D \geq 153.8 \text{ mm}$$

According to maximum strain energy theory :

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{f_y}{F.O.S.}\right)^2$$

$$\Rightarrow \left\{\left[\frac{16}{\pi D^3}\left(M + \sqrt{M^2+T^2}\right)\right]^2 + \left[\frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)\right]^2\right.$$

$$\left. - \frac{2\mu \times 216 \times 16}{\pi^2 D^6}(-T^2)\right\} \le \left(\frac{\sigma_y}{F.O.S.}\right)^2$$

$$\Rightarrow \left(\frac{16}{\pi D^3}\right)^2 \left[\left\{M + \sqrt{M^2+T^2}\right\} + \left\{M - \sqrt{M^2+T^2}\right\} + 2\mu T^2\right] \le \left(\frac{\sigma_y}{F.O.S.}\right)^2$$

$$\Rightarrow M^2 + M^2 + T^2 + M^2 + M^2 + T^2 + 2\mu T^2 \le \left(\frac{\sigma_y}{F.O.S.}\right)^2 \left(\frac{\pi D^3}{16}\right)^2$$

$$\Rightarrow 4M^2 + 2T^2 + 2\mu T^2 \le \left(\frac{\sigma_y}{F.O.S.}\right)^2 \times \left(\frac{\pi^2 D^6}{16 \times 16}\right)$$

Putting the values :

$$4 \times (20 \times 10^6)^2 + (2 + 2 \times 0.3)(40 \times 10^6)^2 \le \left(\frac{250}{2}\right)^2 \frac{(\pi D^3)^2}{16 \times 16}$$

$$D^6 \ge 9.56 \times 10^{12}$$

$$D \ge 145.69 \text{ mm}$$

35. Given data : $B = 300$ mm ; $D = 600$ mm

$A_{st} = 2060$ mm^2 ; $A_{sc} = 804$ mm^2

$d_c = 50$ mm

Effective-Depth $= D - d_c = 600 - 50 = 550$ mm

M20/Fe415

(i) Limiting depth of neutral axis :

$$X_{u_{lim}} = 0.48d = 0.48 \times 550 = 264 \text{ mm}$$

(ii) Actual depth of neutral axis :

Total compressive force = Total tensile force

$$0.36 f_{ck} B x_u + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$$

$$\Rightarrow (0.36 \times 20 \times 300 x_u) + 804(f_{sc} - 0.45 \times 20) = 0.87 \times 415 \times 2060$$

$$\Rightarrow 2160 x_u + 804 f_{sc} = 736527$$

$$x_u = \frac{736527 - 804f_{sc}}{2160}$$

Trail-I : Assuming $f_{sc} = 350$ MPa

$$x_u = \frac{736527 - (804 \times 350)}{2160} = 210.71 \text{ mm}$$

Value of $$\in_{sc} = \frac{0.0035}{x_u}(x_u - d_c)$$

$$\in_{sc} = \frac{0.0035}{210.71}(210.71 - 50) = 0.00267$$

$$f_{sc} \text{ for } 0.00267 = 342 + \frac{(351 - 342)}{(0.00276 - 0.00241)}(0.00267 - 0.00241)$$

$$f_{sc} = 342 + 6.686 = 348.68 \text{ N/mm}^2$$

Trial-II $$x_u = \frac{736527 - (804 \times 348.68)}{2160} = 211.19 \text{ mm}$$

Value of $$\in_{sc} = \frac{0.0035}{211.19}(211.19 - 50) = 0.00267$$

hence $f_{sc} = 348.68$ N/mm^2 and $x_u = 211.19$ mm (adopted)

$\because$ $x_u < x_{u_{lim}} \rightarrow$ Under reinforced

(iii) Moment of resistance :

$$MR = 0.36f_{ck}Bx_u(d - 0.42x_u) + (f_{sc} - 0.45f_{ck})$$
$$A_{sc}(d - d_c)$$

$= \{0.36 \times 20 \times 300 \times 211.19(550 - 0.42 \times 211.19)\} +$
$[(348.68 - 0.45 \times 20) \times 804 \times (550 - 50)]$

$$MR = 346.98 \text{ kN-m}$$

36. ***(i) Most likely Time :***

This is the time required to complete activity if normal conditions prevail. This time estimate lies between the optimistic and the pessimistic time estimates. Thus this time estimate reflects a situation where normal conditions are prevailing, things are as usual and there is nothing exciting. This time is denoted by 't_m'.

(ii) Mean-Time

Mean time t_{avg} (or mean of the distribution) is given by the sum of the time durations t_1, t_2, t_3, ... t_n taken by activities of a particular type for their complition devided by the number of activities n. Thus,

$$t_{avg} = \frac{t_1 + t_2 + t_3 + ... + t_n}{n} = \frac{\Sigma t}{n}$$

(iii) Expected time :

The three time estimates viz the optimistic time t_0, the pessimistic time t_p and the most likely time t_m are identified on the beta probability distribution curve and by using t_p and t_o, the variance and the standard deviation can be calculated. The next step is to objective the average or mean time taken for the complition of an activity is commonly called expected time and is denoted by t_e. If the exact shape of the probability distribution curve is known, the expected time t_e could be accurately calculated.

However, Since the precise curves as generally not available, the use of approximation is made. It has been indicated by the staticians that in beta distribution the average or the expected time is obtained by adding together one sixth of the optimistic time, two thirds of the most likely time and one sixth of the pessimistic time. Thus a large weightage given to most likely time t_m is justified because the chance of complition of the activity in the most likely time t_m is much more than in the optimistic time t_0 or in the the pessimistic time t_0.

37. Given, Width of square footing $\Rightarrow B = 3$ m

Depth of footing $\Rightarrow D_f = 2$ m

Bulk unit weight of sand $\Rightarrow \gamma_t = 18$ kN/m^3

Saturated unit weight of sand $\Rightarrow \gamma_{sat} = 20$ kN/m^3

$$\phi = 35^o \; ; \; N_q = 33 \; ; \; N_\gamma = 34 \text{ and } C = 0$$

We know that for a square footing ultimate bearing capacity is :

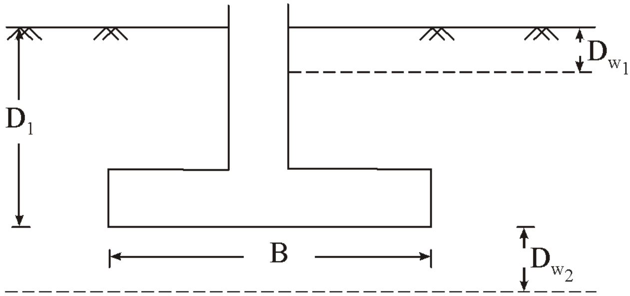
$$q_u = 1.3 \; C \; N_c + \gamma D_f N_q + 0.4\gamma B N_\gamma$$

for soil $C = 0$

$\therefore$ $q_u = \gamma D_f N_q + 0.4\gamma B N_\gamma$

The value of γ in the above equation is susceptible to the location of water table. Thus for each water table position, the ultimate bearing capacity will be different. So, the ultimate bearing capacity will be given as $q_u = \gamma_f D_f N_q + 0.4\gamma_2 B N_\gamma$

(i) When water table is at the ground surface



Now for the given condition $D_{w_1} = 0$ and D_{w_2} is negative so neglected.

$$\gamma_1 = \gamma' + \frac{D_{w_1}}{D_f}(\gamma - \gamma')$$

and $$\gamma_2 = \gamma' + \frac{D_{w_2}}{B}(\gamma - \gamma')$$

$$\gamma_1 = (\gamma_{sat} - \gamma_w) + \frac{0}{2}(\gamma - \gamma')$$

and $$\gamma_2 = \gamma_{sat} - \gamma_w$$

$$\gamma_1 = 20 - 9.81 = 10.19 \text{ kN/m}^3$$

$$\gamma_2 = 20 - 9.81 = 10.19 \text{ kN/m}^3$$

$\therefore$ $$q_u = \gamma_1 D_f N_q + 0.4\gamma_2 B N_\gamma$$

$$= (10.19\times2\times33) + (0.4\times10.19\times3\times34)$$

$$= 1088.29 \text{ kN/m}^2$$

(ii) When the water table is at the footing level

$$D_{w_1} = D_f = 2.0 \text{ m}$$

and $$D_{w_2} = 0$$

$$\gamma_1 = \gamma' + \frac{2}{2}(\gamma - \gamma')$$

and $$\gamma_2 = \gamma' + \frac{0}{3}(\gamma - \gamma')$$

$$\gamma_1 = \gamma' + \gamma - \gamma' = \gamma$$

$$\gamma_2 = \gamma' = \gamma_{sat} - \gamma_w$$

$$\gamma_1 = 18 \text{ kN/m}^3$$

and $$\gamma_2 = 20 - 9.81 = 10.19 \text{ kN/m}^3$$

$$q_u = \gamma_1 D_f N_q + 0.4\gamma_2 B N_\gamma$$

$$q_u = (18\times2\times33) + (0.4\times10.19\times3\times34)$$

$$q_u = 1603.75 \text{ kN/m}^3$$

(iii) When water table is at 1 m below the footing :

$$\gamma_1 = \gamma$$

and $$\gamma_2 = \gamma' + \frac{1}{3}(\gamma - \gamma')$$

⇒ $$\gamma_1 = 18 \text{ kN/m}^3$$

and $$\gamma_2 = \gamma' + \frac{\gamma}{3} - \frac{\gamma'}{3}$$

$$= \frac{2\gamma'}{3} + \frac{\gamma}{3}$$

$$= \frac{1}{3}\left[2\left(\gamma_{sat} - \gamma_w\right) + 18\right]$$

$$= \frac{1}{3}\left[2\left(20 - 9.81\right) + 18\right] = 12.79 \text{ kN/m}^3$$

$$q_u = (18\times2\times33) + (0.4\times12.79\times3\times34)$$

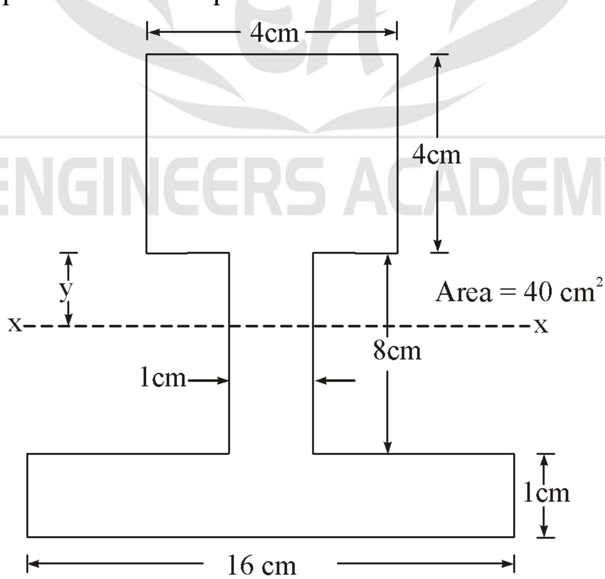
$$q_u = 1703.83 \text{ kN/m}^2$$

38. ***Shape-Factor :*** It is defined as the ratio of the plastic moment and the yield moment of the section. It is a function of cross section form or shape.

Reserve-Strength : It is the strength reserved in a section after initial yielding i.e. yield limit to plastic limit.

It represents the strength due to plastification.

It depends on the shape factor.



Let x-x be equal area axis

$$(4 \times 4) + (1 \times y) = \{(8 - y) \times 1\} + 16 \times 1$$

$$\Rightarrow \quad 16 + y = 8 - y + 16$$

$$2y = 8$$

$$y = 4 \text{ cm}$$

Now, Shape factor $= \dfrac{Z_p}{Z_e}$

and plastic moment capacity $M_p = f_y . z_p$

$$Z_p = \text{Plastic section modulus} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$A = 40 \text{ cm}^2$$

$$\bar{y}_1 = \frac{(16\times 6)+(4\times 2)}{20} = 5.2 \text{ cm}$$

$$\bar{y}_2 = \frac{(16\times\times 1\times 4.5)+(1\times 4\times 2)}{20} = 4 \text{ cm}$$

$$Z_p = \frac{40}{2}(5.2+4) = 20 \times 9.2$$

$$Z_P = 184 \text{ cm}^3$$

∴ Plastic moment capacity

$$\Rightarrow \quad M_p = f_y Z_P = 250 \times 184 \times 10^3$$

$$= 46 \times 10^6 \text{ N-mm} = 46 \text{ kN-m}$$

39. Span = 15 m

B = 300 mm

D = 600 mm

Initial prestressing force P_0 = 1150 kN

Eccentricity at supports = 0

Eccentricity at mid span = 200 mm

$$k_f = 0.15 \text{ per } 100 \text{ m}$$

$$= \frac{0.15}{100} = 15 \times 10^{-4} \text{ /m}$$

$$\mu = 0.35$$

Since the cable profile is parabolic, the equation of profile if

$$y = \frac{4hx}{l^2}(l - x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

$$\Rightarrow \quad \theta(x = 0) = \frac{4h}{l}$$

$$\theta = \frac{4 \times 200}{15000} = 0.0533 \text{ Radians}$$

Assuming that jacking is done at one end only.

Then $\alpha = 2\theta$

$$\alpha = 2 \times 0.0533 = 0.1067 \text{ Radians}$$

$$P_x = P_o e^{-(kx + \mu\alpha)}$$

$$P_x = 1150 \times e^{-(15 \times 10^{-4} 7.5 + 0.35 \times 0.1067)}$$

$$x = \frac{L}{2} = \frac{15}{2} = 7.5 \text{ m}$$

$$P_x = \frac{1150}{e^{0.05}} = 1095.45 \text{ kN}$$

$\therefore$ Loss of prestress $= P_0 - P_x$

$= 1150 - 1095.45 = 54.55$ kN

$$\%\text{Loss} = \frac{54.55}{1150} \times 100 = 4.74\%$$

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